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ON ASYMPTOTICALLY STATISTICAL EQUIVALENT SEQUENCES OF FUNCTIONS

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Abstract: In this paper, we study the concept of asymptotically statistical equivalent sequences with sequences of functions defined on a common domain. The literature provides a deeper understanding of non-negative sequences of real functions in the context of asymptotic analysis. We introduced asymptotically statistical equivalent sequences of functions of multiple α with some novel examples. In addition, certain essential theorems and corollaries are investigated.

Keywords and Phrases: Statistical convergence, asymptotically equivalent sequences, sequence of functions, pointwise convergence.

2020 Mathematics Subject Classification: Primary 40A05, secondary 46A45.

1. Introduction

In 1993, Marouf [6] first gave the theory of asymptotically equivalent sequences of nonnegative numbers and introduced the notion of the behavior of the corresponding terms in two sequences when the sequences increase indefinitely, including the summability method. Later, in 2003, Patterson [11] presented analogous definitions of the study of [6] and merged the concepts with the topic of statistical convergence and presented necessary and sufficient conditions for the matrix summability (for recent developments in the theories related to statistical convergence, one may see [3], [7], [8]). In 2006, Patterson [10] restudied the theory and explored asymptotically lacunary statistical equivalent sequences. The theory being discussed is an open area of ongoing research, several authors have published their findings on the topic from their own perspective in recent years. To capture the evolution of the theory, one may see ([1], [2], [4], [5], [9], [12], [13], [14]).

The current work combines the idea of asymptotic equivalence with the topic of sequences of non-negative functions defined in a common domain. The study examines the behavior of the corresponding terms of the sequence and how they coincide with each other after a certain stage, that is, the difference between them becoming negligible as the sequences spread indefinitely for each variable in the considered domain.

The current study may help us to deal with a sequence that is very difficult to understand directly, but the sequences may be examined by comparing them with other sequences whose terms coincide with the corresponding terms of the original sequence with some multiple α.

In mathematical analysis and other scientific fields both pointwise and uniform convergence of the sequence of functions $< f_n(x) >_{n\geq 1}$ for $x \in D$ play important roles. However, The pointwise convergence of a sequence of functions is the main source of motivation for the current discussion, which focuses on sequences of two nonnegative function at one stage, generalizes the concept of pointwise convergence with asymptotical equivalent sequences of a given multiple, moreover the literature presents geometrical conditions, including some important theorems concerning asymptotically statistically equivalent sequences of functions.

2. Definitions and notations

Definition 2.1. [11] *Two non-negative sequences $u = \langle u_n \rangle_{n\geq 1}$ and $v = \langle v_n \rangle_{n\geq 1}$ are called asymptotically equivalent if*

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1 \qquad (2.1)$$

The case is stated as $u \sim v$.

Definition 2.2. [11] *Two non-negative sequences* $u = \langle u_n \rangle_{n \geq 1}$ *and* $v = \langle v_n \rangle_{n \geq 1}$ *are called asymptotically equivalent of multiple* $\alpha > 0$ *if*

$$\lim_{n \to \infty} \frac{u_n}{v_n} = \alpha \tag{2.2}$$

The case is stated as $u \overset{s_\alpha}{\sim} v$.

Definition 2.3. [3] *The sequence* $u = \langle u_n \rangle_{n \geq 1}$ *is called statistically convergent to limit* l *if for every* $\epsilon > 0$

$$\lim_{n \to \infty} \frac{1}{n} |\{p \leq n : |u_p - l| > \epsilon\}| = 0 \tag{2.3}$$

or $\delta(E) = 0$ *where*

$$E = \{p : |u_p - l| > \epsilon\} \tag{2.4}$$

$\delta(E)$ *is the natural density of the set* E. (for details, see [3])
We now extend the study of ([11]) to sequences of functions and present analogous definitions of the findings of ([11]).

Definition 2.4. *Let* $f = \langle f_n(x) \rangle_{n \geq 1}$ *and* $g = \langle g_n(x) \rangle_{n \geq 1}$ *be two sequences of non-negative function defined on common domain* $D = D_f \cap D_g$, *then the sequences are said to be asymptotically equivalent if*

$$\lim_{n \to \infty} \frac{f_n(x)}{g_n(x)} = 1 \qquad \text{for each} \;\; x \in D \tag{2.5}$$

symbolically, we denote it by $f \sim g$.

Definition 2.5. *Let* $f = \langle f_n(x) \rangle_{n \geq 1}$ *and* $g = \langle g_n(x) \rangle_{n \geq 1}$ *be two sequences of non-negative function defined on common domain* $D = D_f \cap D_g$, *then the sequences are said to be asymptotically equivalent of multiple* α *if*

$$\lim_{n \to \infty} \frac{f_n(x)}{g_n(x)} = \alpha \qquad \text{for each} \;\; x \in D \tag{2.6}$$

symbolically, we denote it by $f \overset{S_\alpha}{\sim} g$.

Example 2.1. Let $f =< f_n(x) >$ where $f_n(x) = x^n - 2nx; \quad x > 1$ & $g =< g_n(x) >; g_n(x) = x^n; \quad x > 1$ then $f \sim g$. Since

$$\lim_{n \to \infty} \frac{f_n(x)}{g_n(x)} = \lim_{n \to \infty} \frac{x^n - 2nx}{x^n} \to 1 - 0 = 1 \quad \text{for every} \quad x > 1$$

Definition 2.6. *Two sequences of functions* $f =< f_n(x) >_{n\geq 1}$ *and* $g =< g_n(x) >_{n\geq 1}$ *are said to be statistically asymptotically equivalent in domain* D *if for every* $\varepsilon > 0$ *& for every* $x \in D$

$$\delta\left\{n \in \mathbb{N} : \left|\frac{f_n}{g_n} - 1\right| > \varepsilon\right\} = 0 \tag{2.7}$$

Symbolically, the case is indicated by $f \overset{st}{\sim} g$.

Example 2.2. Let $f =< f_n(x) >_{n\geq 1}$ and $g =< g_n(x) >_{n\geq 1}$ where

$$f_n(x) = \begin{cases} e^{nx} & ; \quad x > 0 \quad : \quad n \neq k^2 \\ \dfrac{x}{n} & ; \quad n = k^2 \end{cases} \tag{2.8}$$

and

$$g_n(x) = \begin{cases} e^{nx} + nx & ; \quad x > 0 \quad : \quad n \neq k^2 \\ \dfrac{x}{2n+1} & ; \quad n = k^2 \end{cases}$$

for $x > 0$;

$$\lim_{n\to\infty} \frac{f_n(x)}{g_n(x)} = \lim_{n\to\infty} \begin{cases} \dfrac{e^{nx}}{e^{nx} + nx} & ; \quad x > 0 \quad : \quad n \neq k^2 \\ \\ \dfrac{2n+1}{n} & ; \quad n = k^2 \end{cases} \tag{2.9}$$

$$= \begin{cases} 1 & ; \quad n \neq k^2 \\ 2 & ; \quad n = k^2 \end{cases} \tag{2.10}$$

for any $\varepsilon > 0$

$$E = \left\{n \in \mathbb{N} : \left|\frac{f_n}{g_n} - 1\right| > \varepsilon\right\} = \left\{n = k^2 : \left|\frac{f_n}{g_n} - 1\right| > \varepsilon\right\} \text{ and } \delta(E) = 0 \tag{2.11}$$

$$\implies f \sim g.$$

3. Main Results

In this section, we examine the following theorems that are fundamental for further study of the topic. We have established some original theorems and corollaries to show the relationship between the findings.

Theorem 3.1. *If two sequences* $f =< f_n(x) >_{n\geq 1}$ *and* $g =< g_n(x) >_{n\geq 1}$ *of functions defined in some domain* D *are asymptotically equivalent, then* $\lim_{n\to\infty} f_n =$

$\lim_{n\to\infty} g_n$ *in D but the converse need not be true.*

Proof. Given that $f \sim g$, i.e. $\forall\ \varepsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that for every $n > n_0$

$$\left|\frac{f_n}{g_n} - 1\right| < \varepsilon \text{ for every } x \in D \tag{3.1}$$

For $n > n_0$, we have

$$|f_n - g_n| = |g_n|\left|\frac{f_n}{g_n} - 1\right| < |g_n|\,\varepsilon \text{ for every } x \in D \tag{3.2}$$

We now choose $\varepsilon > 0$ such that $\varepsilon < \dfrac{\varepsilon}{|g_n|}$; $(g_n \neq 0)$ then

$$|f_n - g_n| < \varepsilon \text{ for every } x \in D \tag{3.3}$$

$$\implies \lim_{n\to\infty} f_n = \lim_{n\to\infty} g_n \tag{3.4}$$

Conversely,

For the converse part, we consider an example. Let $f_n(x) =< x^n >$; $x \in (0,1)$ and $g_n(x) =< \dfrac{x}{n} >$; $x \in (0,1)$ here, we observe that $\lim_{n\to\infty} f_n = \lim_{n\to\infty} g_n = 0$ but

$$\lim_{n\to\infty} \frac{f_n}{g_n} = \lim_{n\to\infty} \frac{x^n}{\frac{x}{n}} = \lim_{n\to\infty} nx^{n-1} = 0. \tag{3.5}$$

Definition 3.1. *Two non-negative sequences of the function* $f =< f_n(x) >$ *&* $g =< g_n(x) >$ *on a domain D are said to be statistically equivalent of multiple* α*; if for every* $\varepsilon > 0$

$$\delta\left(\left\{n \in \mathbb{N} : \left|\frac{f_n}{g_n} - \alpha\right| > \varepsilon\right\}\right) = 0 \tag{3.6}$$

we denote it by $f \overset{s_\alpha}{\sim} g$.

Theorem 3.2. *For two non-negative sequences of functions* $f =< f_n(x) >$ *and* $g =< g_n(x) >$ *defined on domain D and for two positive scalars* α *and* β *and for each* $x \in D$*; the following are equivalent*

1. $f \overset{s_{\alpha/\beta}}{\sim} g$
2. $f \overset{s_{1/\beta}}{\sim} \alpha g$
3. $\beta f \overset{s}{\sim} \alpha g$

4. $\frac{f}{\alpha} \overset{s}{\sim} \frac{g}{\beta}$

Proof. Let us assume (1) holds, that is, for every $\varepsilon > 0$, and for every $x \in D$

$$\delta\left(\left\{n : \left|\frac{f_n}{g_n} - \frac{\alpha}{\beta}\right| > \varepsilon\right\}\right) = 0 \tag{3.7}$$

Now, $\left\{n : \left|\frac{f_n}{\alpha g_n} - \frac{1}{\beta}\right| > \varepsilon\right\} \subseteq \left\{n : \left|\frac{f_n}{g_n} - \frac{\alpha}{\beta}\right| > \varepsilon\right\}$

Thus (2) holds.

Again, (2) $\implies$ (3)

By (2),

$$\delta\left(\left\{n : \left|\frac{f_n}{\alpha g_n} - \frac{1}{\beta}\right| > \varepsilon\right\}\right) = 0 \tag{3.8}$$

&

$$\left\{n : \left|\frac{f_n}{\alpha g_n} - \frac{1}{\beta}\right| > \varepsilon\right\} \supseteq \left\{n : \left|\frac{\beta f_n}{\alpha g_n} - 1\right| > \beta\varepsilon\right\} \tag{3.9}$$

choose $\varepsilon' = \beta\varepsilon$

$$\implies \delta\left(\left\{n : \left|\frac{\beta f_n}{\alpha g_n} - 1\right| > \varepsilon'\right\}\right) = 0 \tag{3.10}$$

Hence (3) holds.

Again, (3) $\implies$ (4)

Since, for any $\varepsilon > 0$ & for every $x \in D$

$$\left\{n : \left|\frac{\frac{f_n}{\alpha}}{\frac{g_n}{\beta}} - 1\right| > \varepsilon\right\} = \left\{n : \left|\frac{\beta f_n}{\alpha g_n} - 1\right| > \varepsilon\right\} \tag{3.11}$$

Thus (4) holds.

(4) $\implies$ (1) for any $\varepsilon > 0$, we have

$$\left|\frac{\frac{f}{\alpha}}{\frac{g}{\beta}} - 1\right| > \varepsilon \tag{3.12}$$

$$\implies \left|\frac{f_n}{\frac{g}{\beta}} - \alpha\right| > \alpha\varepsilon \quad \text{or} \quad \left|\frac{f_n}{g_n} - \frac{\alpha}{\beta}\right| > \frac{\alpha}{\beta}\varepsilon \tag{3.13}$$

$$\Longrightarrow \left\{ n : \left| \frac{f_n}{g_n} - \frac{\alpha}{\beta} \right| > \frac{\alpha}{\beta}\varepsilon \right\} \subseteq \left\{ n : \left| \frac{\frac{f_n}{\alpha}}{\frac{g_n}{\beta}} - 1 \right| > \varepsilon \right\} \tag{3.14}$$

choose $\varepsilon^{'}$ such that $\frac{\beta}{\alpha}\varepsilon^{'} = \varepsilon$ and it was arbitrary. This implies that for every $\varepsilon^{'} > 0$

$$\delta\left(\left\{ n : \left| \frac{f_n}{g_n} - \frac{\alpha}{\beta} \right| > \varepsilon^{'} \right\} \right) = 0 \tag{3.15}$$

Finally, (1) holds.

Theorem 3.3. *Let $f =< f_n(x) >$, $g =< g_n(x) >$ and $h =< h_n(x) >$ be three non-negative sequences of functions defined on domain D such that $f \overset{s}{\sim} g$ and $g \overset{s}{\sim} h$ then $f \overset{s}{\sim} h$.*

Proof. Since $f \sim g$ this implies that for every $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and for every $x \in D$, we have

$$\delta\left(\left\{ n : \left| \frac{f_n}{g_n} - 1 \right| > \varepsilon_1 \right\} \right) = 0 \tag{3.16}$$

&

$$\delta\left(\left\{ n : \left| \frac{g_n}{h_n} - 1 \right| > \varepsilon_2 \right\} \right) = 0 \tag{3.17}$$

choose $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, then

$$\left\{ n : \left| \frac{f_n}{h_n} - 1 \right| > \varepsilon \right\} \subseteq \left\{ n : \left| \frac{f_n}{g_n} - 1 \right| > \varepsilon_1 \right\} \cup \left\{ n : \left| \frac{g_n}{h_n} - 1 \right| > \varepsilon_2 \right\} \tag{3.18}$$

$$\Longrightarrow \delta\left(\left\{ n : \left| \frac{f_n}{h_n} - 1 \right| > \varepsilon \right\} \right) \leq \delta\left(\left\{ n : \left| \frac{f_n}{g_n} - 1 \right| > \varepsilon_1 \right\} \right) + \delta\left(\left\{ n : \left| \frac{g_n}{h_n} - 1 \right| > \varepsilon_2 \right\} \right) = 0 \tag{3.19}$$

This yields

$$f \overset{s}{\sim} h. \tag{3.20}$$

Theorem 3.4. *Let $f =< f_n(x) >$, $g =< g_n(x) >_{n\geq 1}$ and $h =< h_n(x) >_{n\geq 1}$ be non-negative sequences of functions defined in D such that $f \overset{s_\alpha}{\sim} g$ and $g \overset{s_\beta}{\sim} h$ then for any two positive real numbers α and β; $f \overset{s_{\alpha\beta}}{\sim} h$.*

Proof. We are given that $f \overset{s_\alpha}{\sim} g$ implies that $f \sim \alpha g$ and $g \overset{s_\beta}{\sim} h$ implies that

$g \sim \beta h$ (see theorem 3.2) for any positive scalar α, we have $\alpha g \sim \alpha\beta h$. (By the previous theorem 3.3), $f \sim \alpha g$ and $\alpha g \sim \alpha\beta h$.
Hence $f \sim \alpha\beta h$ this gives $f \overset{s_{\alpha\beta}}{\sim} h$.

Theorem 3.5. *If $f =< f_n(x) >$, $g =< g_n(x) >$ and $h =< h_n(x) >$ are non-negative sequences of functions defined on a common domain D. If for two real positive scalars α and β, $f \overset{s_\alpha}{\sim} h$ and $g \overset{s_\beta}{\sim} h$ then $f + g \overset{s_{\alpha+\beta}}{\sim} h$.*
Proof. We have $f \overset{s_\alpha}{\sim} h$ and $g \overset{s_\beta}{\sim} h$ this implies that for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ and for each $x \in D$

$$\delta\left(\left\{n : \left|\frac{f_n}{h_n} - \alpha\right| > \varepsilon_1\right\}\right) = 0 \tag{3.21}$$

and

$$\delta\left(\left\{n : \left|\frac{g_n}{h_n} - \beta\right| > \varepsilon_2\right\}\right) = 0 \tag{3.22}$$

choose $\varepsilon = min\{\varepsilon_1, \varepsilon_2\}$ then

$$\left\{n : \left|\frac{f_n + g_n}{h_n} - (\alpha+\beta)\right| > \varepsilon\right\} \subseteq \left\{n : \left|\frac{f_n}{g_n} - \alpha\right| > \varepsilon_1\right\} \cup \left\{n : \left|\frac{g_n}{h_n} - \beta\right| > \varepsilon_2\right\} \tag{3.23}$$

$$\implies \delta\left(\left\{n : \left|\frac{f_n + g_n}{h_n} - (\alpha+\beta)\right| > \varepsilon\right\}\right) \leq \delta\left(\left\{n : \left|\frac{f_n}{g_n} - \alpha\right| > \varepsilon_1\right\}\right) + \delta\left(\left\{n : \left|\frac{g_n}{h_n} - \beta\right| > \varepsilon_2\right\}\right) \tag{3.24}$$

$$\implies \delta\left(\left\{n : \left|\frac{f_n + g_n}{h_n} - (\alpha+\beta)\right| > \varepsilon\right\}\right) = 0 \tag{3.25}$$

$\implies f + g \overset{s_{\alpha+\beta}}{\sim} h.$

Theorem 3.6. *If $f =< f_n(x) >_{n\geq 1}$ and $g =< g_n(x) >$ are the two positive and bounded sequences of functions defined in a common domain D such that for a positive scalar α; $f \overset{s_\alpha}{\sim} g$ then $\frac{1}{f} \overset{s_{1/\alpha}}{\sim} \frac{1}{g}$.*
Proof. Given, $f \overset{s_\alpha}{\sim} g$ therefore for every $\varepsilon > 0$ and for each $x \in D$

$$\delta\left(\left\{n : \left|\frac{f_n}{g_n} - \alpha\right| > \varepsilon\right\}\right) = 0 \tag{3.26}$$

Now, for $\varepsilon_1 > 0$

$$\left\{n : \left|\frac{g_n}{f_n} - \frac{1}{\alpha}\right| > \varepsilon_1\right\} = \left\{n : \left|\frac{f_n}{g_n} - \alpha\right| > \frac{\alpha\epsilon_1|f_n|}{|g_n|}\right\} \tag{3.27}$$

Since the sequences $< f_n >$ and $< g_n >$ are bounded in their domain, we can therefore find the numbers M_1 and M_2 such that $|f_n| \leq M_1$ and $|g_n| \leq M_2$ for each x. We choose $\varepsilon > 0$ such that $\varepsilon < \dfrac{\varepsilon_1|\alpha|M_1}{M_2}$ then for arbitrary ε_1, we have

$$\left\{n : \left|\frac{g_n}{f_n} - \frac{1}{\alpha}\right| > \varepsilon_1\right\} \subseteq \left\{n : \left|\frac{f_n}{g_n} - \alpha\right| > \varepsilon\right\} \tag{3.28}$$

$$\implies \delta\left(\left\{n : \left|\frac{g_n}{f_n} - \frac{1}{\alpha}\right| > \varepsilon_1\right\}\right) = 0 \tag{3.29}$$

Hence, $\dfrac{1}{f} \overset{s_{1/\alpha}}{\sim} \dfrac{1}{g}$.

Corollary 3.1. *If* $f_k =< f_n^k(x) >$; $k = 1, 2, 3, ...p$ *and* $h =< h_n(x) >$ *are non-negative sequences of functions defined in a common domain D such that for positive scalars* α_k*;* $k = 1, 2, 3, ...p$ $f_k \overset{s_{\alpha_k}}{\sim} h$ *then* $\sum_{k=1}^{p} f_k \overset{s_\alpha}{\sim} h$ *; where* $\alpha = \sum_{k=1}^{p} \alpha_k$.

Proof. We use the induction method to prove it
By **"Theorem 3.5"** we have

$$f_1 + f_2 \overset{s_{\alpha_1+\alpha_2}}{\sim} h \tag{3.30}$$

Let this be true for $k = m$, then

$$f = f_1 + f_2 + f_3 + ... + f_m \overset{s_{\alpha'}}{\sim} h \tag{3.31}$$

where $\alpha' = \alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_m$

Now, for $k = m + 1$,

$$f + f_{m+1} = \overset{s_{\alpha'+\alpha_{m+1}}}{\sim} h \tag{3.32}$$

$$\implies \sum_{k=1}^{p} f_k \overset{s_\alpha}{\sim} h; \quad \alpha = \sum_{k=1}^{p} \alpha_k.$$

Corollary 3.2. *If* $f_k =< f_n^k(x) >$; $k = 1, 2, 3, ...p$ *be the 'p', non-negative sequences of functions defined in a common domain D, such that for positive scalars* α_k*;* $k = 1, 2, 3, ...(p - 1)$*;*

$f_j \overset{s_{\alpha_j}}{\sim} f_{j+1}$*;* $\quad j = 1, 2, 3, ...(p - 1)$ *then* $f_1 \overset{s_\alpha}{\sim} f_p$ *where* $\alpha = \prod_{j=1}^{p-1} \alpha_j$

Proof. The proof of this corollary is a direct consequence of the **"Theorem 3.4"** (which can also be proved using the induction method).

4. Conclusion

We have presented analogous definitions with some original results of the definitions explored in [6]. We also analyzed the original theorems and corollaries, including primary results in the context of sequences of functions defined on a common domain. Our discoveries generalized the theory of asymptotically statistical equivalent sequences for sequences of functions that are essential for the continued study of the concepts discussed in [6], [9], [11]).

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